Scientia Africana, Vol. 12 (No.2), December 2013. Pp19-22
 ISSN 1118 – 1931

STABILIZING THE BORDER STEADY-STATE SOLUTION OF TWO INTERACTING POPULATIONS

[1]N. M. Nafo and [2]E. N. Ekaka-a

[1]Department of Mathematics and Computer Science,
Rivers State University of Science and Technology, Port Harcourt, Nigeria

[2]Department of Mathematics and Statistics,
University of Port Harcourt, Port Harcourt, Nigeria

Received: 22-11-12
Accepted:05-12-12

ABSTRACT
The interaction between two legumes such as cowpea and groundnut depends on the tool of mathematical modelling which uses a system of continuous non-linear first order ordinary differential equations. In this paper, we have successfully developed a feedback control which has been used to stabilize an unstable steady-state solution (0, 3.3534). This convergence has occurred when the values of the final time are 190, 200, 210 and 220 which corresponds to the scenario when the value of the step length of our simulation is 0.01 and the initial data are (4, 10). The details of our present contribution are presented and discussed.

Key words: Best-fit parameters, agricultural data, 1-norm, 2-norm, infinity-norm.

INTRODUCTION
In our previous study (Yan and Ekaka-a 2011), we have selected the precise values of the deterministic competition model between two legumes of cowpea and groundnut over a growing season in days (Ekpo and Nkanang 2010). For this system of continuous nonlinear first order ordinary differential equations with model parameters such as $a = 0.0225$, $b = 0.006902$, $c = 0.0005$, $d = 0.0446$, $e = 0.01$ and $f = 0.0133$, it is very clear that this system of model equations has four steady-state solutions namely (0, 0), (0, 3.3534), (3.2599, 0) and (3.1908, 0.9543). The trivial steady-state solution is unstable because its calculated eigenvalues has two positive eigenvalues of 0.0225 and 0.0446 whereas the steady-state solution (0, 3.3534) is clearly unstable because its eigenvalues are −0.0446 and 0.0208. The steady-state solution (3.2599, 0) is unstable having two eigen-values of −0.0.0225 and 0.0120. The only unique positive steady-state solution 3.1908, 0.9543 is stable because its eigenvalues are -0.0234 and -0.0113. The numerical challenge at this level of analysis is to investigate the process and extent of stabilizing the three unstable steady-state solutions as well as attempting to stabilize the only stable steady-state solution. The stabilization method will be defined and discussed.

The aim of this paper is to stabilize a nonlinear system of first order ordinary differential equations of the form

$$(2.1) \quad \frac{dN_1(t)}{dt} = F(N_1(t), N_2(t)).$$

$$(2.2) \quad \frac{dN_2(t)}{dt} = G(N_1(t), N_2(t)).$$

with the initial conditions $N_1 = N_{10} > 0$ and $N_2 = N_{20} > 0$.

MATERIALS AND METHODS

The arbitrary steady-state solution (N_{1e},N_{2e}) is unstable, that is, the point (N_1,N_2) is not convergent to (N_{1e},N_{2e}) when t tends to infinity. To stabilize the following solution we did the following [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [13] and [14], the process of stabilizing a mathematical model of a population system is conducted using three standard procedures; finding the linearized problem about (N_{1e},N_{2e}), finding a positive definite matrix Pi from the Riccati equation and applying the Pi matrix in the nonlinear equation to check if (N_1,N_2) is convergent to (N_{1e},N_{2e}). The next stage in our algorithm is implemented following these steps: putting the steady-state solution (N_{1e},N_{2e}) which we want to stabilize; choosing m = 0 for the unstable case and m = 1 for the stable case; choosing different initial values for a different steady-state solution if this choice is realistic; choosing a different final time *Tfinal* for a different steady-state solution and choose a time step k; and choosing the number of loops. We chose a feedback control; solved the nonlinear system and constructed the subplots which would show the convergence behaviour of the uncontrolled and controlled solution trajectories.

By using this defined algorithm, we have been able to stabilize the unstable steady-state solutions for this system of model equations. Our contributions are presented and discussed below.

DISCUSSION OF RESULTS

In this section, we present and discuss the stabilization of the border steady- state solution (0, 3.3534). We attempt for the first time to study the extent of stabilizing (0, 3.3534) in which the cowpea legume is driven into extinction while the groundnut legume will survive at its carrying capacity value of 3.3534.

For this simulation, we have considered the initial data (4, 10) and the step length k = 0.01. Our analysis has revealed that for this choice of simulation parameters, the steady-state solution (0, 3.3534) can only be fully stabilized when the values of the final time are 190, 200, 210, and 220. These novel contributions are displayed in the table below:

Scientia Africana, Vol. 12 (No.2), December 2013. Pp19-22
 ISSN 1118 – 1931

Table 1: Convergence of the steady-state solution (0, 3.3534)

Examples	Stabilization of (0, 3.3534)		
no	*Tfinal*	N_{1e}	N_{2e}
1	10	0.0519	4.3962
2	20	0.0281	3.9197
3	30	0.0160	3.6768
4	40	0.0094	3.5430
5	50	0.0056	3.4663
6	60	0.0033	3.4212
7	70	0.0020	3.3943
8	80	0.0012	3.3781
9	90	0.0007	3.3684
10	100	0.0004	3.3625
11	110	0.0003	3.3567
12	120	0.0002	3.3554
13	130	0.0001	3.3546
14	140	0.0001	3.3541
15	150	0.000036479	3.3538
16	160	0.000021891	3.3537
17	170	0.000013020	3.3536
18	180	0.0000076264	3.3535
19	190	0.0000043463	3.3534
20	200	0.0000023516	3.3534
21	210	0.0000011386	3.3534
22	220	0.00000040091	3.3534

In this paper, our major contribution is the key fact that the steady-state solution (0, 3.3534) can be considered as being clearly stabilized when the final time ranges from 190 weeks to 220 weeks. As far as we know, this stabilization strategy will provide useful insights in the function of the ecosystem and stability of these two interacting legumes especially in the events of expected global warming indicators and other unknown ecological perturbations.

REFERENCES

Damgaard, C. Evolutionary Ecology of Plant-Plant Interactions-An Empirical Modelling Approach, Aarthus University Press, 2004.

Ekaka-a, E.N. Computational and Mathematical Modelling of Plant Species Interactions in a Harsh Climate, PhD Thesis, Department of Mathematics, The University of Liverpool and The University of Chester, United Kingdom, 2009.

Ekaka-a, E.N. and Nafo, N.M. (2012).Stabilising a Mathematical Model of Stock Market Population System.*Scientia Africana*. Vol. II (No 1). June 2012, pp 92-97

Ekpo, M.A. and Nkanang, A.J. Nitrogen fixing capacity of legumes and their Rhizosphereal microflora in diesel oil polluted soil in the tropics, *Journal of Petroleum and Gas Engineering* 1(4), (2010), pp. 76-83.

Goudriaan, J. Monteith, J.LA mathematical function for crop growth based on light interception and leaf area expansion, *Annals of Botany* 66, (1990), pp. 695-701.

Goudriaan, J. van Laar, H.H. Modelling potential crop growth processes, Dordrecht: Kluwer Academic Publishers , 1994.

Hunt, R. Studies in Biology, no. 96: Plant Growth Analysis, London: Edward Arnold (Publishers) Limited , 1981.

Ibia, T.O. Ekpoand M.A. Inyang, L.D. Soil Characterisation, Plant Diseases and Microbial Survey in Gas Flaring Community in Nigeria, *World J. Biotechnol.* 3, (2002), pp. 443-453.

Kot, M. Elements of Mathematical Ecology, Cambridge University Press, 2001.

Murray, J.D. Mathematical Biology, 2nd Edition Springer Berlin, 1993.

Renshaw, E. Modelling Biological Populations in Space and Time, Cambridge University Press,1991.

Tilman, D. Dynamics and Structure of Plant Communities, Princeton University Press, 1988.

Yan, Y. and Ekaka-a, E.N (2011).Stabilising a Mathematical Model of Population System.*Journal of the Franklin Institute*. Vol. 348, Issue 10, December 2011, pp 2744 – 2758

Yin, X. Goudriaan, J. Lantinga, E.A. Vos J. and Spiertz, H.J.A Flexible Sigmoid Function of Determinate Growth, *Annals of Botany* 91, (2003), pp. 361-371.